Quantum 521: TEST # 2

You have 24 hours from the time of reading this exam to submit your work. No book, no notes, no computers, no google, no discussion. Calculator OK.

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possibly useful info:

Time dependence of expectation value for arbitrary state $|\Psi(t)\rangle$ in the Schrödinger picture

$$\frac{d}{dt}\langle\hat{A}\rangle = \frac{i}{\hbar}\langle[\hat{H},\hat{A}]\rangle + \langle\frac{\partial\hat{A}}{\partial t}\rangle$$

The Pauli spin matricies:

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Rotation of spinor about $\hat{n}$ direction by an angle ϕ:

$$\hat{R}(\phi\hat{n}) = \cos\left(\frac{\phi}{2}\right)\hat{I} - i\vec{\sigma}\cdot\hat{n}\sin\left(\frac{\phi}{2}\right);$$

Formula for angular momentum raising and lowering operators,

$$J_{\pm}|j,m\rangle = \hbar\sqrt{j(j+1)-m(m\pm 1)}|j,m\pm 1\rangle$$

For the simple harmonic oscillator define the constants $x_0 = \sqrt{\frac{\hbar}{m\omega}}$ and $p_0 = \sqrt{\hbar m\omega}$. The Hamiltonian is

$$H = \hbar\omega\left(N + \frac{1}{2}\right)$$

where $N = a^\dagger a$,

$$\frac{x}{x_0} = \frac{1}{\sqrt{2}}(a^\dagger + a)$$

$$\frac{p}{p_0} = i\frac{1}{\sqrt{2}}(a^\dagger - a)$$

and the commutators $\left[a, a^\dagger\right] = 1$, $[N, a] = -a$, $\left[N, a^\dagger\right] = a^\dagger$

#1) In the Heisenberg picture, the time dependance goes into the operator, $A^H = U^\dagger(t) A U(t)$ and expectation values are $\langle\psi(t)|A|\psi(t)\rangle = \langle\psi(0)|A^H|\psi(0)\rangle$. The operators satisfy the Heisenberg equations of motion,

$$\frac{dA^H}{dt} = \frac{i}{\hbar}\left[H, A^H\right]$$

a) For the simple harmonic oscillator, show that the the Heisenberg equations of motion for the operators X(t) and P(t) are the same as the classical equations of motion.

b) Find the operator $X(t)$.

c) Evaluate the correlation function $c(t) = \langle X(t)X(0)\rangle$ for the ground state. Note that it is an amplitude, not a probability so that it need not be real.

#2) For the a particle in an angular momentum eigenstate $|\ell, m\rangle$, calculate $\langle L_x\rangle$ $\langle L_y\rangle$, $\langle L_x^2\rangle$ $\langle L_y^2\rangle$

#3) Consider a particle subject to a constant force equal to f in one dimension. Show that the momentum space propagator is

$$\langle p, t|p', 0\rangle = \delta\left(p - p' - ft\right)\exp\left(\frac{i({p'}^3 - p^3)}{6m\hbar f}\right)$$

Normalize the momentum space energy eigenfunctions such that $\langle E|E'\rangle = \delta(E|E')$. Note that E is not restricted to be positive.